Week 5: Mass-spring loudspeaker model, acoustic domain, domain coupling

Microphone and Loudspeaker Design - Level 5

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What are we covering today?

1. Mass-spring model of loudspeaker
2. Acoustic domain
3. Acoustic analogues
4. Transducers
5. Ideal transformers
6. Domain coupling
7. Tutorial questions

A weekly fact about Salford..!

Did you know...

- In 1806, Chapel Street became the first street in the world (not Pall Mall! [1807]) to be lit by gas.

Mass-spring model of loudspeaker

Mass-spring-damper: equation of motion

- We have analysed the dynamics of a mass-spring-damper system using an equivalent circuit approach
- Now we will consider a more conventional approach based on laws of classical mechanics
- Newton's 2nd Law:

$$\sum_i F_i = Ma = \frac{d^2x}{dt^2} \tag{1}$$

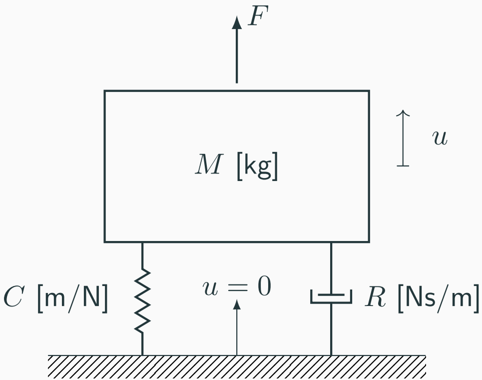

Figure 1: Mass-spring-damper.

$$-kx - R\frac{dx}{dt} + F_{ext} = M\frac{d^2x}{dt^2} \quad \rightarrow \quad F_{ext} = kx + R\frac{dx}{dt} + M\frac{d^2x}{dt^2} \tag{2}$$

Equation of motion: complete solution

- The general solution is the complementary function plus the particular integral:

$$x(t) = x_p + x_{cf} \tag{3}$$

$$x(t) = \frac{F_0 e^{j\omega t}}{j\omega \left(R + j\left[\omega M - \frac{k}{\omega}\right]\right)} + e^{-\frac{R}{2M}t}\left[A_1 e^{\frac{\sqrt{R^2-4Mk}}{2M}t} + A_2 e^{\frac{-\sqrt{R^2-4Mk}}{2M}t}\right] \tag{4}$$

- As $t \to \infty$ the transient part of the solution tends to zero.
- **We will focus on the steady state part.**

Q factor vs. oscillation

$$x_{cf} = e^{-\frac{R}{2M}t}\left[A_1 e^{\frac{\sqrt{R^2-4Mk}}{2M}t} + A_2 e^{\frac{-\sqrt{R^2-4Mk}}{2M}t}\right] \quad (5)$$

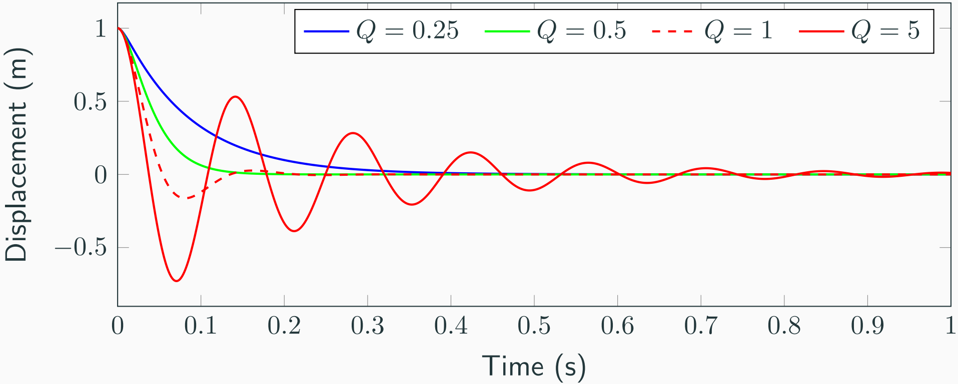

Figure 2: Over, under and critically damped oscillation - $x(0) = 1$ and $\dot{x}(0) = 0$

Q factor vs. peakyness

$$u = \frac{F}{R + j\omega m + \frac{k}{j\omega}} = \frac{F}{\frac{\sqrt{mk}}{Q} + j\omega m + \frac{\omega_c^2 m}{j\omega}} = \frac{F}{\frac{\sqrt{mk}}{Q} + j\omega m\left(1 - \frac{\omega_c^2}{\omega^2}\right)} \tag{6}$$

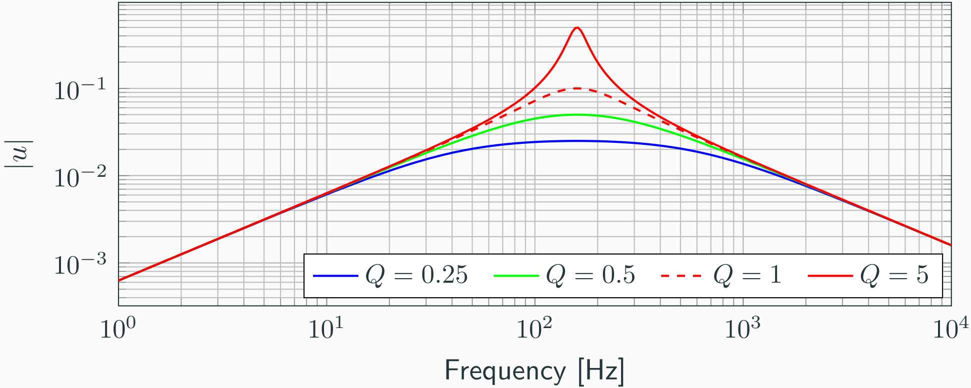

Figure 3: Frequency response of over, under and critically damped mass-spring-damper.

Acoustic domain

Acoustic impedance: volume velocity

- The volume velocity is the product of the component of particle velocity u normal to a vibrating surface and the differential surface area:

$$dU = \hat{n} \cdot u dS \tag{7}$$

- For a uniformly vibrating surface area S we have

$$U = uS \tag{8}$$

- Has units of [m^3/s] hence the name *volume* velocity
- Volume velocity is a scalar (not a vector like particle velocity)

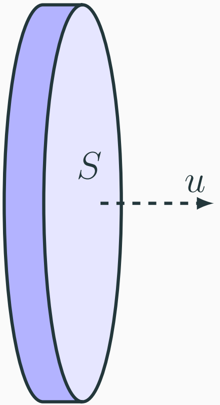

Figure 4: Volume velocity of a rigid piston.

Acoustic impedance: relation to mechanical impedance

- Acoustic impedance

$$Z_A = \frac{p}{U} \qquad U \text{ is the volume velocity} \tag{9}$$

- Mechanical impedance

$$Z_M = \frac{F}{u} \qquad u \text{ is the surface velocity} \tag{10}$$

- Recalling that $p = F/S$ and $U = uS$

$$Z_A = \frac{p}{uS} = \frac{F/S}{uS} = \frac{F}{uS^2} = \frac{Z_M}{S^2} \tag{11}$$

- Acoustic and mechanical impedance are related by factor of $1/S^2$

Acoustic elements: mass

- Like any other mass, a mass of air is governed by Newton's 2nd Law:

$$F = M_M \frac{du}{dt} \tag{12}$$

- In the acoustic domain we tend to deal with pressure $p = F/S$ and volume velocity $U = uS$

$$\frac{F}{S} = \frac{M_M}{S} \frac{d(uS)}{dt} \frac{1}{S} \rightarrow p = M_A \frac{dU}{dt} = j\omega M_A U \tag{13}$$

- We define the **acoustic mass** $M_A = M_M/S^2$

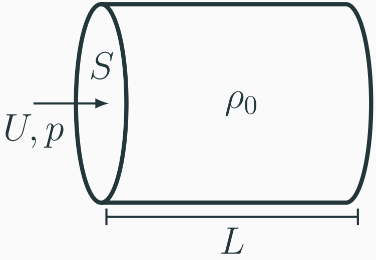

Figure 5: Acoustic mass.

$$Z_{AM} = \frac{p}{U} = j\omega M_A$$

Acoustic elements: mass

- What happens to the mass as the diameter of the element increases?
- The acoustic mass is,

$$M_A = \frac{M_M}{S^2} = \frac{LS\rho_0}{S^2} = \frac{L\rho_0}{S} \qquad (14)$$

- Inversely proportional to area!
- If you want something to have a smaller acoustic mass you can either *reduce* the length or *increase* the diameter! This is a little counter-intuitive...

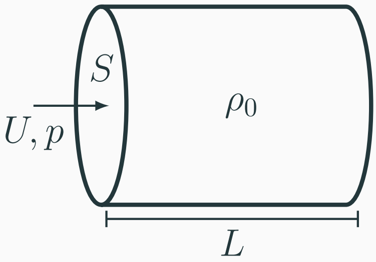

Figure 3: Acoustic mass.

$$Z_{AM} = \frac{p}{U} = j\omega M_A$$

Acoustic elements: cavity/spring

- The equation governing the compression of a volume of air by a net force is:

$$F = \frac{1}{C_M} \int u dt \tag{15}$$

- Converting to acoustic units

$$\frac{F}{S} = \frac{1}{C_M S} \int \frac{uS}{S} dt \rightarrow p = \frac{1}{C_A} \int U dt = \frac{1}{j\omega C_A} U \tag{16}$$

- Where the **acoustic compliance** $C_A = C_M S^2$

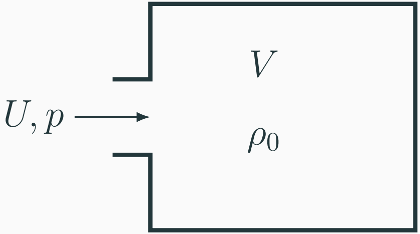

Figure 4: Acoustic compliance.

$$Z_{AC} = \frac{p}{U} = \frac{1}{j\omega C_A}$$

Acoustic elements: cavity/spring

- Acoustic compliance is related to the volume of air and its properties

$$C_A = \frac{V}{\rho_0 c^2} \tag{17}$$

- The larger the volume, the more compliant the cavity (easier to compress)

- **Remember:** compliance is inverse stiffness

$$C = \frac{1}{k} \tag{18}$$

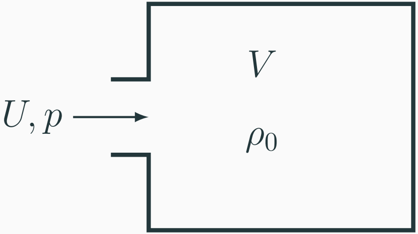

Figure 4: Acoustic compliance.

$$Z_{AC} = \frac{p}{U} = \frac{1}{j\omega C_A}$$

Acoustic elements: resistance

- The acoustic resistance through a fine mesh is governed by:

$$F = R_M u \tag{19}$$

- Converting to acoustic units

$$\frac{F}{S} = \frac{R}{S}\frac{uS}{S} \rightarrow p = R_A U \tag{20}$$

- Where the **acoustic resistance** $R_A = R_M / S^2$

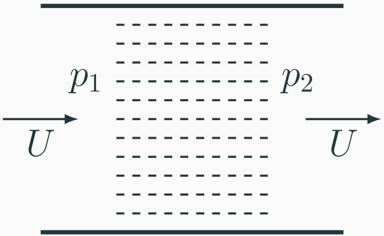

Figure 5: Acoustic compliance.

$$Z_{AR} = \frac{p}{U} = R_A$$

Acoustic analogues

Acoustic domain: impedance analogy

- Acoustic impedance

$$Z_A = \frac{p}{U} \tag{21}$$

- Electrical impedance

$$Z_E = \frac{V}{I} \tag{22}$$

- To preserve the notion of impedance we must have that

$$p \to V \qquad U \to I \qquad Z_A \to Z_E \tag{23}$$

- What electrical components do the acoustic components resemble?

$$Z_{AM} = j\omega M_A \qquad Z_{AC} = \frac{1}{j\omega C_A} \qquad Z_{AR} = R_A \tag{24}$$

Acoustic domain: impedance analogy

Element	Impedance analogy	Mobility analogy
Mass	Mass $\leftrightarrow$ Inductor $Z_A = j\omega M_A \leftrightarrow Z_E = j\omega L_E$	
Cavity	Cavity $\leftrightarrow$ Capacitor $Z_A = \frac{1}{j\omega C_A} \leftrightarrow Z_E = \frac{1}{j\omega C_E}$	
Resistance	Resistance $\rightarrow$ Resistor $Z_A = R_A \leftrightarrow Z_E = R_E$	

Acoustic domain: mobility analogy

- **Acoustic mobility**

$$Y_A = \frac{1}{Z_A} = \frac{U}{p} \tag{25}$$

- Electrical impedance

$$Z_E = \frac{V}{I} \tag{26}$$

- According to the mobility analogy we must have that

$$p \rightarrow I \qquad U \rightarrow V \qquad Y_A \rightarrow Z_E \tag{27}$$

- What electrical components do the acoustic components resemble?

$$Y_{AM} = \frac{1}{j\omega M_A} \qquad Y_{AC} = j\omega C_A \qquad Y_{AR} = \frac{1}{R_A} \tag{28}$$

Impedance/mobility analogies: summary

Element	Impedance analogy	Mobility analogy
Mass	Mass ↔ Inductor $Z_A = j\omega M_A \leftrightarrow Z_E = j\omega L$	Mass ↔ Capacitor $\frac{1}{Z_A} = \frac{1}{j\omega M_A} \leftrightarrow Z_E = \frac{1}{j\omega C_E}$
Cavity	Cavity ↔ Capacitor $Z_A = \frac{1}{j\omega C_A} \leftrightarrow Z_E = \frac{1}{j\omega C_E}$	Cavity ↔ Inductor $\frac{1}{Z_A} = j\omega C_A \leftrightarrow Z_E = j\omega L_E$
Resistance	Resistance ↔ Resistor $Z_A = R_A \leftrightarrow Z_E = R_E$	Resistance ↔ Resistor $\frac{1}{Z_A} = \frac{1}{R_A S} \leftrightarrow Z_E = R_E$

Acoustic domain: impedance and mobility analogy

- For the **impedance analogy** we made the following equivalences:
  - Pressure as being analogous to voltage $p \to V$ (drop parameter)
  - Volume velocity as being analogous to current $U \to I$ (flow parameter)

- For the **mobility analogy** we make the following equivalences:
  - Pressure as being analogous to current $p \to I$ (flow parameter)
  - Volume velocity as being analogous to voltage $U \to V$ (drop parameter)

Constructing an equivalent acoustic circuit: impedance analogy

- The mass, mesh and cavity have the same volume velocity.
- Recall the definition of **impedance analogy**:

$$p \to V \qquad U \to I \tag{27}$$

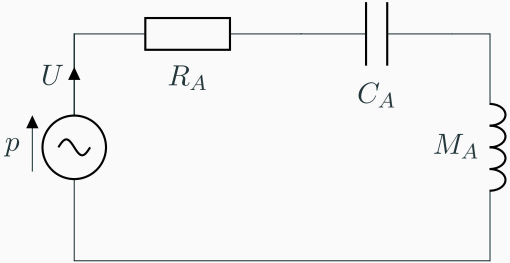

Figure 6: Equivalent circuit.

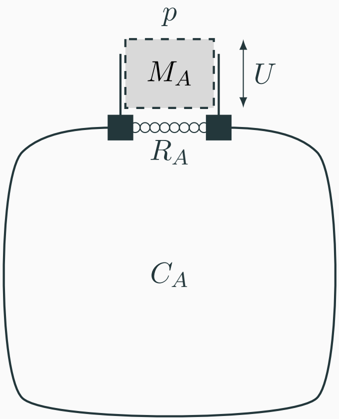

Figure 6: Helmholtz resonator.

Constructing an equivalent acoustic circuit: mobility analogy

- The mass, mesh and cavity have the same volume velocity.
- Recall the definition of **mobility analogy**:

$$p \to I \qquad U \to V \tag{28}$$

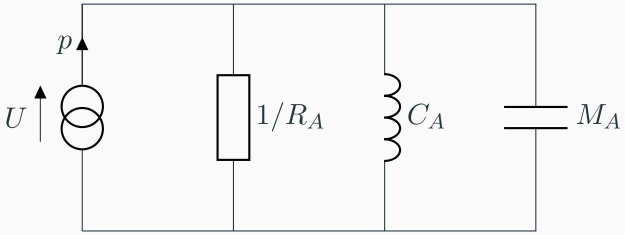

Figure 8: Equivalent circuit.

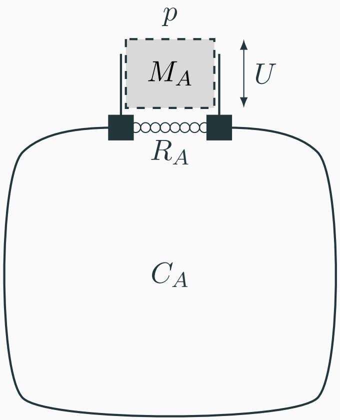

Figure 6: Helmholtz resonator.

Equivalent circuits: summary

- Can use equivalent circuits to describe mechanical and acoustic systems as electrical circuits.
- Useful when we connect everything together...
- Two different analogies:
  - **Impedance**: mechanical $F \to V$, $u \to I$; acoustical $p \to V$, $U \to I$; retain analogy between impedances, but lose topology
  - **Mobility**: mechanical $F \to I$, $u \to V$; acoustical $p \to I$, $U \to V$; lose analogy between impedances, but retain topology
- So far we have derived the equivalent circuits for a mass-spring-damper, and a Helmholtz resonator
- **Still to consider: how to combine domains...**

Transducers

Transducers

- Transducers are devices that convert one form of energy to another, i.e. they couple domains together.
- Two important types:
  1. Electro-mechanic: couple electrical domain to mechanical domain
  2. Mechano-acoustic: couple mechanical domain to acoustical domain

Transducers: mechano-acoustic

- The equations that couple the mechanical and acoustic domains are:

$$F = pS \tag{29}$$

$$U = uS \tag{30}$$

- The first relates pressure to force
- The second relates volume velocity to surface velocity

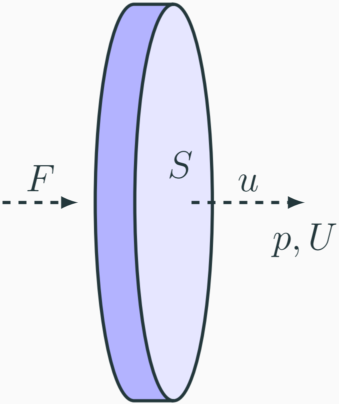

Figure 9: Mechano-acoustic transduction

Transducers: electro-mechanic

- We will focus on **electrodynamic transducers**
- Key principle:
  - Conductor carrying a current (voice coil) in a magnetic field is subject to the Lorentz force
- The Lorentz force for a coil of wire of length L in a magnetic field of strength B through which a current I passes, is given by

$$F = BLI \tag{31}$$

- But, electro-dynamic transduction is a two way phenomena...

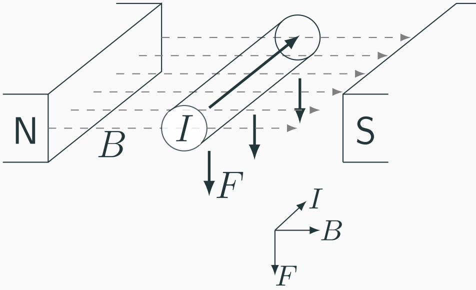

Figure 10: Lorentz force on a charge carrying conductor

Transducers: electro-mechanic

- When a conductor moves in a magnetic field, a voltage is generated across its length

$$V = BLu \tag{32}$$

- The voltage V is often called the back EMF (electro-motive force); it opposes the flow of current reducing the overall current

$$I = I_{applied} - I_{bEMF} \tag{33}$$

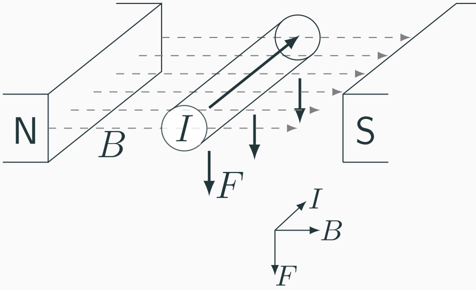

Figure 10: Lorentz force on a charge carrying conductor

Transducers: summary

- Electro-mechanical transduction:

$$F = BLI \tag{34}$$

$$u = \frac{V}{BL} \tag{35}$$

- Mechano-acoustical transduction:

$$p = \frac{F}{S} \tag{36}$$

$$U = Su \tag{37}$$

- The equations happen to look very similar to the equations of an ideal transformer...

Ideal transformers

Ideal transformers

- Transformers are passive electrical devices that transfer electrical energy between two or more circuits.
- Their fundamental operation is based on Faraday's law of induction:
  - A time varying current around the primary coil induces a time varying magnetic field within the transformer core.
  - A secondary coil responds to the time varying magnetic field by inducing a voltage across its length.

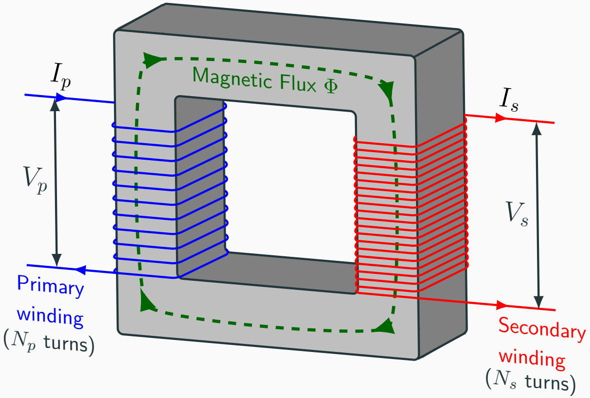

Figure 11: Ideal transformer

Ideal transformers

- An ideal transformer is a theoretical linear transformer that is lossless and perfectly coupled
  - Power input = power output
- Electrical power: $P = IV$

$$P_{in} = P_{out} \rightarrow V_p I_p = V_s I_s \quad (38)$$

$$\frac{V_p}{V_s} = \frac{I_s}{I_p} \quad (39)$$

- Scaling of voltage and current is reciprocal - increased voltage decreased current

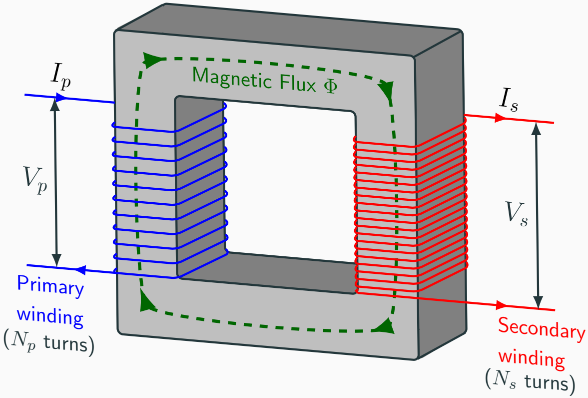

Figure 11: Ideal transformer

Ideal transformers

- The turns ratio α determines voltage change

$$\frac{V_p}{V_s} = \frac{N_p}{N_s} = \alpha \tag{40}$$

- Reciprocal turns ratio determines effect on current

$$\frac{I_p}{I_s} = \frac{N_s}{N_p} = \frac{1}{\alpha} \tag{41}$$

- Gives us two equations:

$$V_p = \alpha V_s \qquad I_p = \frac{1}{\alpha} I_s \tag{42}$$

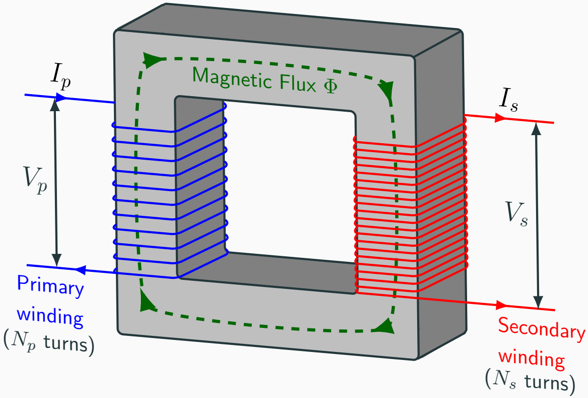

Figure 11: Ideal transformer

Ideal transformers

- Ideal transformer

$$V_p = \alpha V_s \qquad I_p = \frac{1}{\alpha} I_s \qquad (43)$$

- Electro-mechanic transduction

$$V = BLu \qquad I = \frac{1}{BL} F \qquad (44)$$

- Mechano-acoustic transduction

$$u = \frac{1}{S} U \qquad F = Sp \qquad (45)$$

- Look pretty similar don't they... $\alpha = ??$

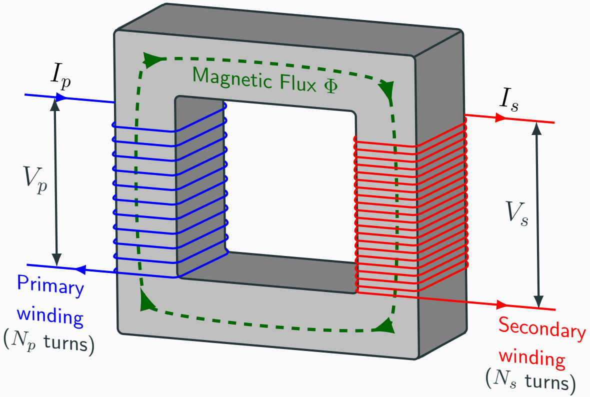

Figure 11: Ideal transformer

Domain coupling

Domain coupling by ideal transformers: electro-mechanical

- Ideal transformer vs. electro-mechanical transduction with turns ratio: $\alpha = BL$

$$V_p = \alpha V_s \longleftrightarrow V = BLu \qquad I_p = \frac{1}{\alpha} I_s \longleftrightarrow I = \frac{1}{BL} F \tag{46}$$

- Secondary side: $u \sim V$, $F \sim I$ (what analogy is this?)

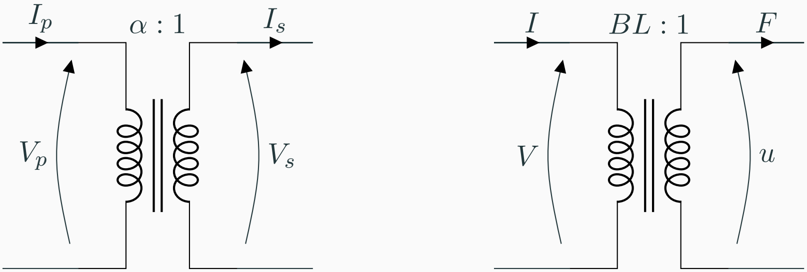

Figure 12: Ideal transformer vs. electro-mechanical coupling.

Domain coupling by ideal transformers: electro-mechanical

- Secondary side: $u \sim V$, $F \sim I$ - **mobility analogy**
- Substitute in our equivalent mobility-based circuit for a mass-spring-damper
- Resistor and inductor model the properties of the voice coil

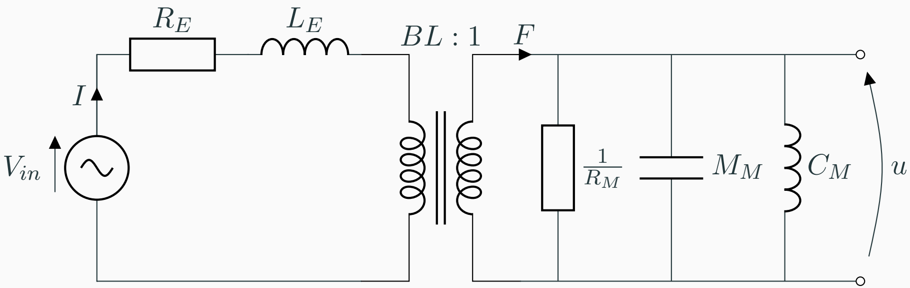

Figure 13: Ideal transformer coupling between electrical and mechanical domains.

Domain coupling by ideal transformers: mechano-acoustical

- Ideal transformer vs. mechano-acoustical transduction with turns ratio: $\alpha = 1/S$

$$V_p = \alpha V_s \longleftrightarrow u = \frac{1}{S}U \qquad I_p = \frac{1}{\alpha}I_s \longleftrightarrow F = Sp \tag{47}$$

- Secondary side: $U \sim V$, $p \sim I$ (what analogy is this?)

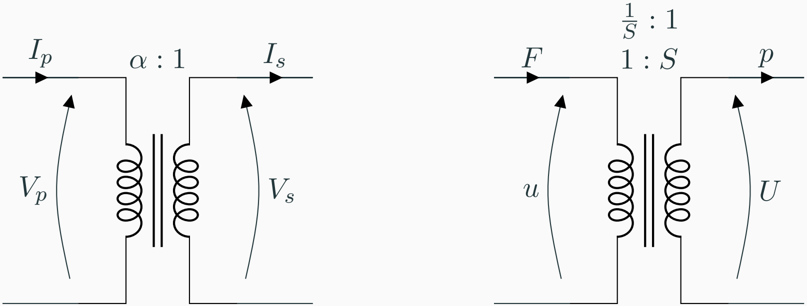

Figure 14: Ideal transformer vs. electro-mechanical coupling.

Domain coupling by ideal transformers: mechano-acoustical

- Secondary side: $U \sim V$, $p \sim I$ - **mobility analogy**
- This is all looking good... but currently a loudspeaker in a vacuum.
- What about acoustic loading? (a job for next week)

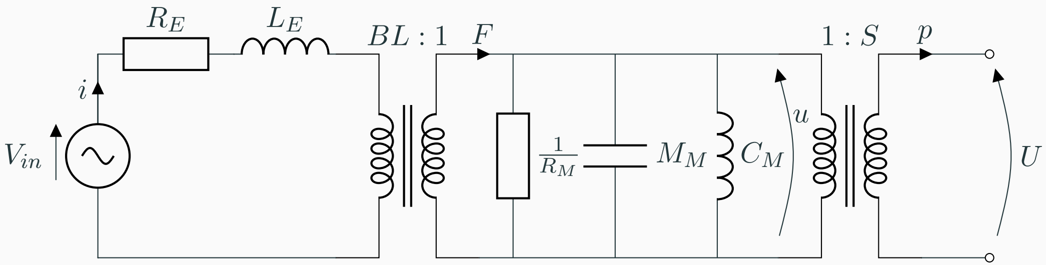

Figure 15: Ideal transformer coupling between electrical and mechanical domains.

Next week...

- Removing transformers
- Acoustic loading
- Acoustic radiation (cone velocity to acoustic pressure)

- Reading:
  - Acoustic domain: lecture notes, chp. 4 (all)
  - Coupling domains: lecture notes, sec. 6.1-6.5 (all)